Error Reduction in Cave Conduit Modeling

Daniel P. Callahan
Senior Integrative Exercise
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CONTENTS

Abstract

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Abstract

Volumetric models of cave systems are often used in hydrogeologic studies in karst terrains. One means of generating these models is through the use of cave survey data which describe the accessible extent of the cave, as well as the rough dimensions—defined by only four points—of cross sections along the way. These dimensions are often taken in a manner that does not accurately describe the cave. Previous studies place median error around ±10%. This study examines the development of a software tool to analyze passage cross sections and propose techniques to significantly reduce error. In a study of 18 individual cross sections, the software found a means of measuring each that would reduce calculated error to effectively 0% based on sketches of the cross section. Furthermore, it was found that for the 18 passages studied, the four points should most likely be interpreted as defining the length and width of a rectangle, as opposed to the vertices of a quadrilateral or the axes of an ellipse.

Keywords: karst, cave, survey, lrud, error

Introduction

Karst aquifers, characterized by a capacity for self-development and self-organization, often develop in carbonate strata as a consequence of the hydrogeologic cycle (Klimchouk and Ford, 2000). Such aquifers, when unconfined, host at least 90% of the world's explored caves (Palmer, 1991; Worthington et al., 2000). As karst terrains represent up to 20% of the Earth's land area (Palmer, 1991), any study of the caves therein may grant insight into major hydrogeologic and speleogenetic processes.

Karst can form and operate both near the surface, and in deep-seated environments. In the latter case, the structure exists without any surface representation or input (Klimchouk and Ford, 2000), while in the former case, there is interaction as "nearly all major surface karst features owe their origin to internal drainage, subsidence, and collapse triggered by the development of underlying caves" (Palmer, 1991). As near-surface karst acts as a rapid sink for incoming water, karst terrains often exhibit depressed soil productivity, limited groundwater availability, reduced groundwater quality, and an abundance of subsidence features. These effects can be highly local—"dramatic differences in the functioning of karst landscapes exist across small distances" (Aley, 2000). Accurate portrayals of the subsurface and its caves are required for the development of responsible land use practices.

Accurate surveys of caves in karst regions are thus crucial for both land management and scientific purposes. Most commonly, caves are measured by establishing a series of points ("survey stations") throughout the explored extent of the cave. These are referenced to each other by three metrics: distance, azimuth, and inclination, all taken while facing into the cave. At least one station is located outside and defined with the assistance of a GPS receiver. These data alone are sufficient to produce a "stick map," which traces the explored "spine" of the cave from station to station. In order to produce a proper plan-view or three-dimensional map of the cave, the location of its walls relative to its spine must be determined. This is often accomplished with just four measurements at each survey station: The distance Left, Right, Up, and Down from the station to the cave wall. These data are often abbreviated "LRUD," and can be used to create a series of rough cross sections of the cave at each survey station. It's then possible to create an approximate volumetric model of the cave by integrating between each cross section.

Passage cross sections can be used to calculate paleodischarge, to infer speleogenetic controls on cave inception and morphology, and to estimate a given system's fluid storage volume (Sasowsky and Bishop, 2006). However, the jump from LRUD to cross section is not straightforward, largely due to the limitations of representing complex passage morphologies with only four points in space. In 2006, Sasowsky and Bishop conducted an "empirical study of conduit radial cross-section determination and representation methods," ultimately concluding that the selection of a method by which to convert LRUD data into two dimensional sections did significantly affect subsequent analyses of the cave.

For instance, three immediate options (Fig. 1) present themselves: A quadrilateral connecting all the points as vertices, a rectangle of area $(\text{L} + \text{R}) \times (\text{U} + \text{D})$, and an ellipse of area $\pi \times \frac{\text{L}+\text{R}}{2} \times \frac{\text{U}+\text{D}}{2}$. Each representation will give different areas for a given set of LRUD values, and thus each representation will produce a different median error in a given cave.

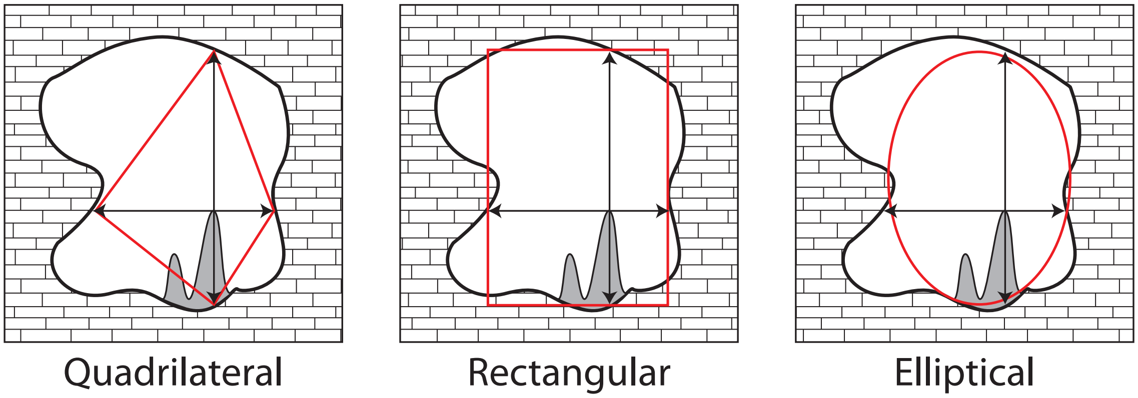

Figure 1: Possible methods of determining cave passage cross sections. Cave surveyors generally take four measurements, known as LRUD data (Left, Right, Up, Down), at each survey station. These four points are then used to create a polygon that roughly approximates the shape of the passage. Figure adapted from Sasowsky and Bishop (2006).

Sasowsky and Bishop (2006) also examined the difference between laser-measured and visually-estimated LRUD data and found that surveyors tended to overestimate by an average of 26% for all measurements regardless of the actual distance. While "grossly accurate," such estimates were judged "poor for analytical use, and far below typical survey standards." Thus, for a survey to be analytically useful, it must employ manual measurements of all distances. This mandate is especially problematic when surveying fully-saturated cave systems, as the amount of time that can be spent collecting data is limited by the amount of breathable gas carried by the divers (Kincaid, 2000). Due to this constraint, most LRUD data from underwater surveys are estimated.

When accurate LRUD measurements were taken, quadrilateral, rectangular, and elliptical representations yielded median errors of -45%, -11%, and +10% respectively. In this instance, interpreting the survey's LRUD data as a series of quadrilaterals would grossly misrepresent the cave. However, this is only necessarily true for this particular cave and survey station layout. The way to best represent an arbitrary passage is still an open question, and one which was anticipated by Sasowsky and Bishop (2006):

> "Because conduit shape results from such factors as lithology, structure, and hydrologic history, it follows that certain caves will be more accurately represented by [a given strategy]. It might be possible in future work to quantify this effect by studying errors present in passages of different rock types, hydrologic origin, etc."

This paper explores means of improving the accuracy of cave surveys in light of Sasowsky and Bishop's findings.

METHODS

Two variables govern the accuracy of a given interpretation of LRUD data: The origin of the measurements, and the method of passage representation. Sasowsky and Bishop (2006) only addressed the latter, as they used fixed station locations from a previous survey. In order to examine both variables concurrently, a computer program, "OptimLRUD," was developed to exhaustively analyze images or sketches of arbitrary cross sections.

OptimLRUD works by reading a black and white image of a passage's cross section. In the image, each white pixel represents one square unit of void space, while non-white pixels represent the surrounding bedrock. The total number of white pixels is taken as the actual cross sectional area of the sketched passage. OptimLRUD then iterates over the void space, calculating LRUD data from the perspective of each point in the plane of the cross section. This data is used to find cross sectional areas with each of the three representations: quadrilateral, rectangular, and elliptical.

By using the calculated data, OptimLRUD finds the magnitude of error for each of the three representations at every point on the sketch. This data is then presented as a both a raw database, and as a surface with colors representing different error magnitudes. Overlaying the surface onto the original image reveals the measurement locations that produce the greatest accuracy, while the database allows for a statistical analysis of the suitability of each representation.

The utility of this application is tested by applying it to the original data collected by Sasowsky and Bishop (2006). As in their study, the laser-based 16-point measurements are considered to be the actual cross sectional area. OptimLRUD is used to analyze each of Sasowsky and Bishop's 18 stations, with the results being compared to those found in the that study.

RESULTS

Sasowsky and Bishop Dataset

Figure 2 shows cross sections of each survey station examined by Sasowsky and Bishop (2006). For the purposes of this study, the area defined by the 16 laser measurements is considered to be the actual area of the passage.

The distribution of error for each interpretation is presented as Figure 3. The average of each station's median error was -60%, -20%, and -37% respectively for quadrilateral, rectangular, and elliptical representations. The average of each station's minimum absolute error was 29%, 0%, and 3% for each respective method.

As seen in Figure 3, in no case did a non-outlying quadrilateral measurement fall within 10% of the actual measurement. Rectangular interpretations, on the other hand, produced measurements indistinguishable from the actual value for every single station, with 12 of those measurements falling between the set's median and third quartile. Elliptical representations produced 12 values

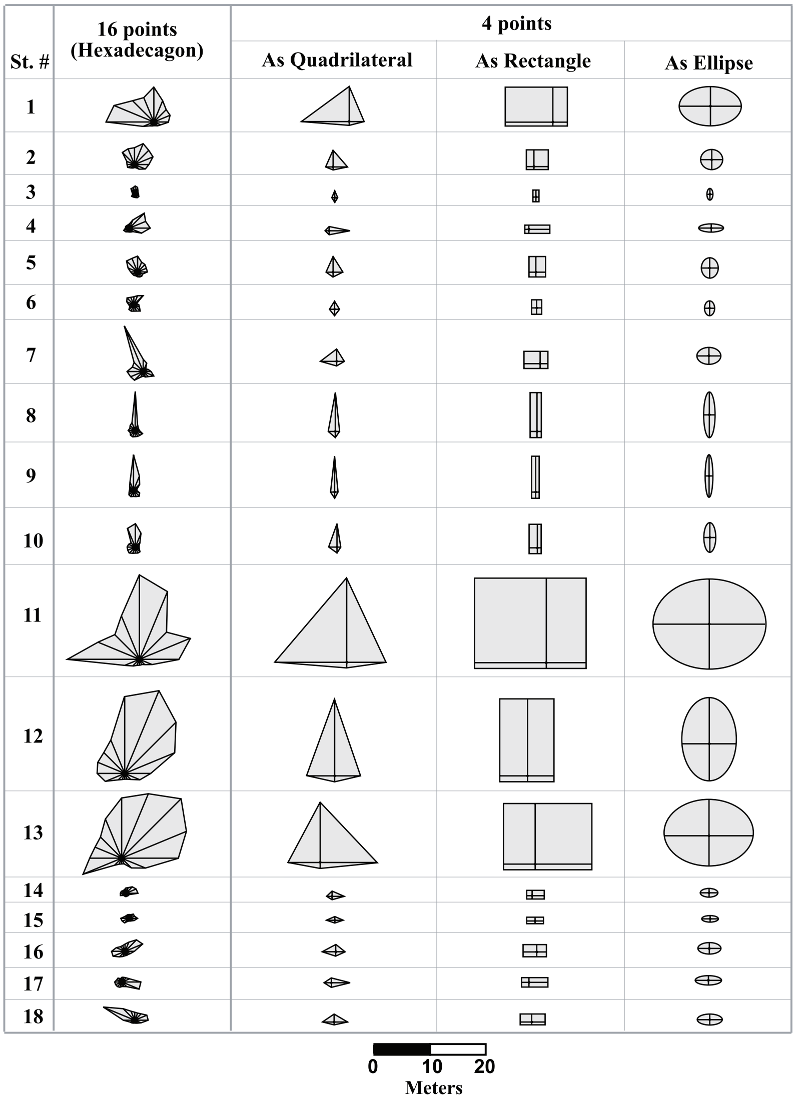

Figure 2: A list of Sasowsky and Bishop's 18 survey stations in Scott Hollow Cave, indicating the origin of the measurements, as well as quadrilateral, rectangular, and elliptical representations of each passage. Figure adapted from Sasowsky and Bishop (2006).

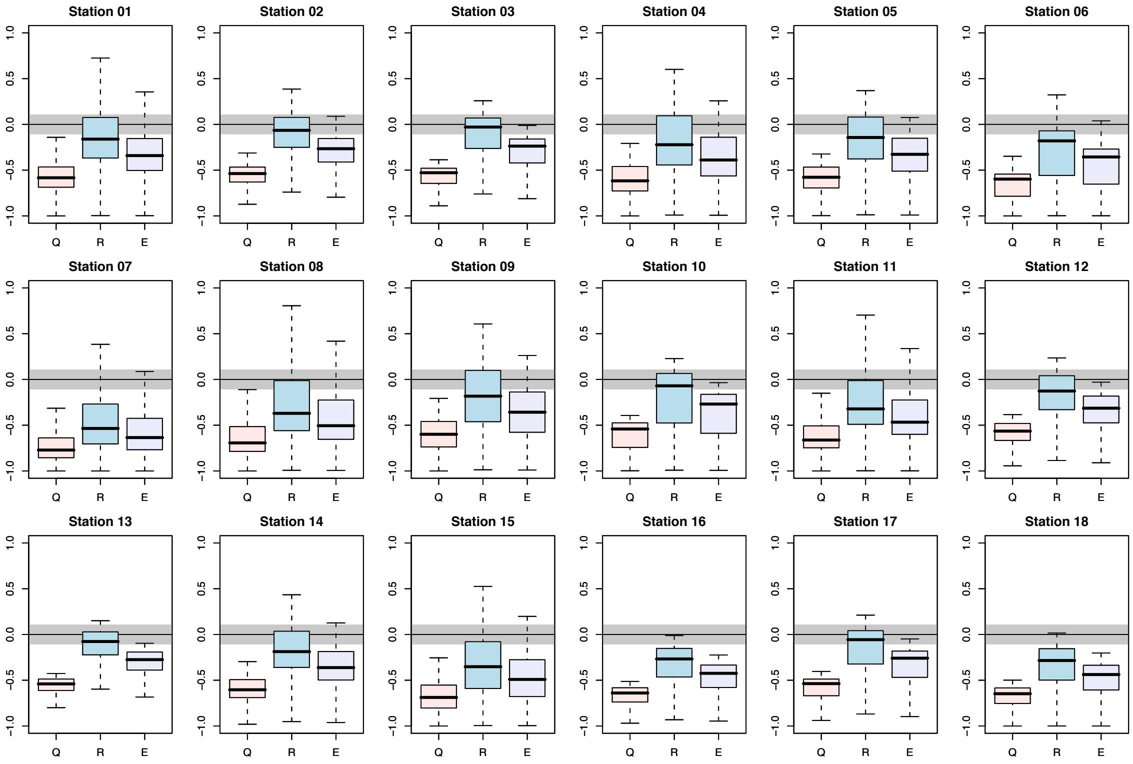

Figure 3: A visual summary of the distribution of error for each representation method (Q: Quadrilateral, R: Rectangular, E: Elliptical) across each passage. Outliers are not shown. The shaded bar demarcates the interval from -10% to +10%, which was found by Sasowsky and Bishop (2006) to be the median error for rectangular and elliptical representations of cave passages in Scott Hollow Cave. The presence of data points inside the shaded region for every single station indicates the potential to significantly outperform current surveying methods.

that fell at 0% error, but each was well above the third quartile. Furthermore, the third quartile always occurred below the -10% error threshold for elliptical representations.

Tolerance for Sketch Inaccuracy

OptimLRUD also analyzed a set of deformed images which were derived from a traced photograph of a karst keyhole (Fig. 4). All of the resulting surfaces indicated peaks in accuracy in the same relative locations, even when the sketch was reduced to its most basic elements—a circle and a rectangle. This resilience was present with all three representations.

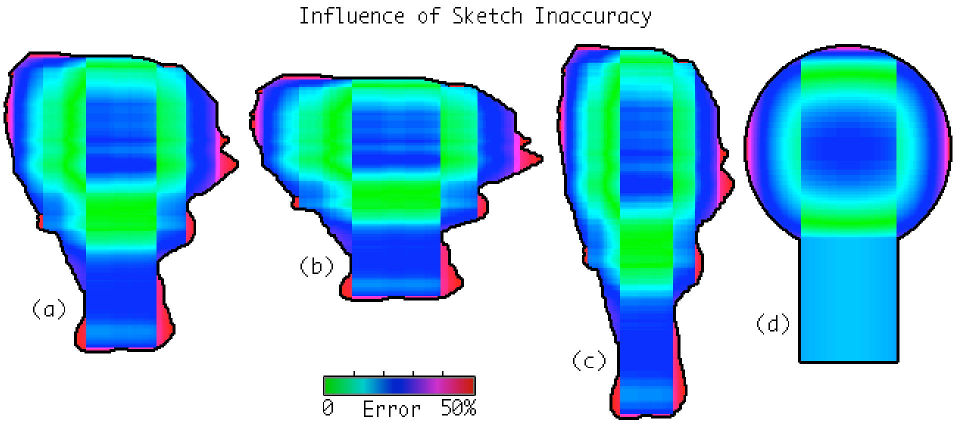

Figure 4: Handling of distorted input sketches. Image (a) was traced from a photograph of a "keyhole" passage, and then distorted and reduced to yield subfigures b, c, and d. The consistent relative distribution of error indicates that the method that OptimLRUD employs is robust and allows decomposing caves into basic morphologic components. Only the rectangular surfaces are shown; other methods exhibited similarly minimal alteration.

Discussion and Conclusions

The survey studied by Sasowsky and Bishop outperformed the median error in the data that OptimLRUD generated (-45/-11/+10% vs. -60/-20/-37%) regardless of representation. This indicates that the surveyors were using a reasonable heuristic to determine survey station placement. However, the average minimum errors that OptimLRUD found (29/0/3%) are far superior to the error in the original study. Thus, it should possible to reliably outperform current surveying techniques by including OptimLRUD in pre-survey planning. It's visual output, shown in Figure 5, is generated quickly and is easy to understand. Green areas in OptimLRUD's output indicate survey station placements that would reduce absolute error to less than 8%. From these data, it's clear that quadrilateral models are not appropriate for representing the cross sectional area of the studied passages.

This method's resilience to inaccuracies in sketches (Fig. 4) suggests that a library of common passage morphologies could be compiled and processed en masse to produce field guides of best practices in various types of caves. This goal would be aided by the fact that caves have prototypical morphologies based on their speleogenetic environment.

The resilience also ensures OptimLRUD does not require perfectly representative sketches in

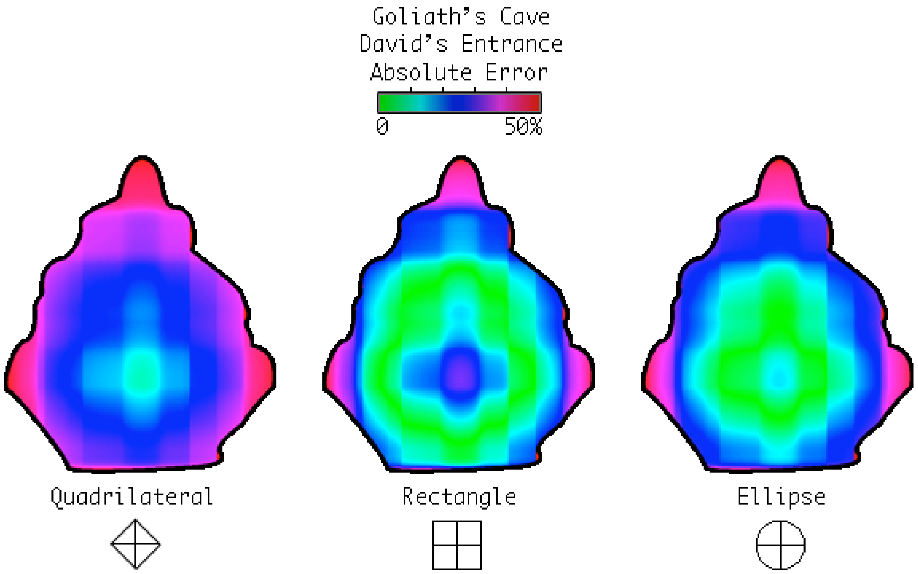

Figure 5: Annotated composite output from OptimLRUD for a passage in Goliath's Cave, Fillmore County, Minn. Ideal survey station placement and representation methods are immediately apparent; any station located in a green region will outperform the median survey error in Sasowsky and Bishop's study.

order to be of use; a surveyor could quickly move through the areas of interest, note their general morphology, and check them with OptimLRUD before the full survey commences. The software could also be adapted to run on a handheld computer for interactive use during a survey.

While OptimLRUD is certainly useful *a priori*, it does have applications to completed surveys. For instance, OptimLRUD can be used as an auditing tool. This can be accomplished by taking a laser rangefinder to a handful of survey stations, obtaining high quality representations of the cross sections, and then examining the current station placement in the context of OptimLRUD's suggestions. However, most significantly, the same process can be applied in order to determine a probable margin of error, which can then be corrected without the requiring a system to be resurveyed.

This project is foundational in addressing Sasowsky and Bishop's call for quantification of "errors present in passages of different rock types, hydrologic origin, etc," and provides a direct means by which to significantly reduce error in past and future surveys. Furthermore, the result that rectangular representations can achieve nearly zero error in every passage studied suggests that, unlike Sasowsky and Bishop's conclusion, eight-point surveys may not be necessary at all.

One concern with approaching a survey with OptimLRUD is that it may suggest placing survey stations in mid-air, where there are no physical markers to sight or reference. The simplest solution is to conduct a standard stick map survey, but also record a separate "LRUD origin" at each station. The LRUD origin could be defined relative to the classic survey station with the aid of a protractor and fiberglass tape. The decoupling of each survey station's duties would ensure that the best possible locations are selected for representing both the accessible extent of the cave and its general volumetric properties.

Nevertheless, by intelligently selecting a point from which to take LRUD measurements, and a manner by which to represent them, the accuracy of cave surveys can be significantly improved. And with improved surveys come improved paleodischarge and fluid storage calculations, as well as an improved understanding of speleogenetic controls on cave inception and morphology.

Acknowledgements

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Appendix A: Source Code for OptimLRUD

OptimLRUD was developed using the Python programming language, version 2.5, and the Python Imaging Library (PIL) version 1.1.6. It’s source code is as follows:

```
#!/bin/env python
”””
OptimLRUD Analytical Cave Survey Tool
Copyright (c) 2008 Dan Callahan

This program is released under the terms of the MIT License.
”””

from __future__ import division
from os.path import basename, splitext
import Image, ImageColor
import numpy
import math
import csv

def analyze(imgPath):
    ”””Analyze an image and write data files to the current folder.”””
    im = Image.open(imgPath)
    lruds = getLruds(im)

    funcs = (areaQuad, areaRect, areaElip)
    for func in funcs:
        base = ’-’.join((splitext(basename(imgPath))[0],
                         func.__name__[4:].lower()))
        csvname = ’’.join((base, ”.csv”))
        imname = ’’.join((base, ”.png”))

        errs = getErrors(lruds, func)

        csv.writer(open(csvname, ”wb”)).writerows(errs)
        paintError(im.copy(), errs).save(imname)

def getLR(px, x, y):
    ”””Calculate Left and Right values from a given image and origin”””
    l, r = x, x
    while px[l-1, y] == (255, 255, 255):
        l -= 1
    while px[r+1, y] == (255, 255, 255):
        r += 1
    return (l, r)

def getUD(px, x, y):
```

```
    """Calculate Up and Down values from a given image and origin"""
    u, d = y, y
    while px[x, u-1] == (255, 255, 255):
        u -= 1
    while px[x, d+1] == (255, 255, 255):
        d += 1
    return (u, d)

def areaQuad(x, y, l, r, u, d):
    """Find quadrilateral area for a given origin and set of LRUD measures"""
    # NE + SE + SW + NW - Overlap
    return float(\
    (.5 * (r - x + 1) * (y - u + 1)) + \
    (.5 * (r - x + 1) * (d - y + 1)) + \
    (.5 * (x - l + 1) * (d - y + 1)) + \
    (.5 * (x - l + 1) * (y - u + 1)) - \
    (r - l + 1) - (d - u + 1))

def areaRect(x, y, l, r, u, d):
    """Find rectangular area for a given origin and set of LRUD measures"""
    # Length * Width)
    return float((d - u + 1) * (r - l + 1))

def areaElip(x, y, l, r, u, d):
    """Find elliptical area for a given origin and set of LRUD measures"""
    # pi*X*Y where X and Y are semimajor/semiminor axes
    return float(math.pi * ((r - l + 1) / 2) * ((d - u + 1) / 2))

def getLruds(im):
    """Find LRUD values for every white pixel in a given image"""
    px = im.load()
    a = numpy.zeros((im.size[0], im.size[1], 6), int)
    results = []

    # Make two passes over the image, one caching LR values, the other UD.
    # Direct array access and assignment profiles faster than using slices.
    # LR Scan
    l, r = None, None
    for y in xrange(im.size[1]):
        l, r = None, None
        for x in xrange(im.size[0]):
            if px[x,y] == (255, 255, 255):
                if not(l and r):
                    l, r = getLR(px, x, y)

                a[x,y,0] = x
                a[x,y,1] = y
```

```
                a[x,y,2] = l
                a[x,y,3] = r
            else:
                l, r = None, None
    # UD Scan
    u, d = None, None
    for x in xrange(im.size[0]):
        u, d = None, None
        for y in xrange(im.size[1]):
            if px[x,y] == (255, 255, 255):
                if not(u and d):
                    u, d = getUD(px, x, y)

                a[x,y,4] = u
                a[x,y,5] = d

                results.append(a[x,y].tolist())
            else:
                u, d = None, None
    return results

def getErrors(lruds, areaFunc):
    ”””Find area calculation errors for a given data set and function”””
    area = len(lruds)
    results = []
    for (x, y, l, r, u, d) in lruds:
        results.append((x, y, (areaFunc(x, y, l, r, u, d)/area) - 1))
    return results

def getErrorColors():
    ”””Returns a list of discrete colors for representing error”””
    colors = []
    for i in xrange(241): # 241 = range from 120 (green) to 360 (red)
        colors.append(ImageColor.getrgb(”hsl(%i,100%%,50%%)” % (120 + i)))
    return colors

def paintError(im, errlist):
    ”””Paints a heatmap of error values onto an image”””
    px = im.load()
    colors = getErrorColors()
    numcolors = len(colors) - 1
    for (x, y, e) in errlist:
        if abs(e) > 1:
            px[x,y] = (255,0,0)
        else:
            px[x,y] = colors[int(numcolors * abs(e))]
    return im
```